Indian Statistical Institute, Bangalore
B. Math (II), Second Semester 2018-19
MidSemester Examination : Statistics (II)

Date: 28-02-2019 **Maximum Score 40** **Duration: 3 Hours**

1. Pareto distribution that works well as income distribution is specified by the following *probability density function (pdf)*

$$f(x|\alpha,\theta) = \theta\alpha^{\theta}x^{-\theta-1}I_{(\alpha,\infty)}(x);\ \alpha > 0 \text{ and } \theta > 0$$

Let $X_1,\ X_2, ..., X_n$ be a random sample from the above distribution. Find *method of moments* (MOM) estimators for α and θ.

[8]

2. A discrete model that is often used for the waiting time X to failure of an item is given by the *pmf*

$f(k|\theta) = \theta^{(k-1)}(1-\theta);\ k = 1, 2, ...,;\ 0 < \theta < 1.$

Suppose that we only record the time of failure, if the failure occurs on or before r and otherwise just note that the item has lived at least $r+1$ periods. Let Y denote this censored waiting time. Write down the *pmf* of Y. If $Y_1,\ Y_2, ..., Y_n$ is a random sample from this censored waiting time distribution, obtain an *mle* for θ. Does your *mle* of θ agree with $\frac{T-n}{T-M}$, where $T = \sum_{i=1}^{n} Y_i$ and $M =$ no. of indices i such that $Y_i = r+1$.

[10]

3. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from the distribution specified by

$$f(x|\theta,\lambda) = \frac{1}{\lambda}e^{-\frac{(x-\theta)}{\lambda}}I_{(\theta,\,\infty)}(x);\ x,\theta \in \mathbb{R},\ \lambda > 0.$$

Obtain a minimal sufficient statistic for (θ, λ).

[12]

4. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from uniform distribution on $(\theta, \theta+1)$. Derive the distribution of the range $R = X_{(n)} - X_{(1)}$. Hence or otherwise prove that it is an ancillary statistic.

[8]

5. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from uniform distribution on $(\theta, \theta+1)$. Is $\left(X_{(1)}, X_{(n)}\right)$ a complete statistic? Substantiate.

[8]